Balanced triangulations on few vertices and an implementation of cross-flips

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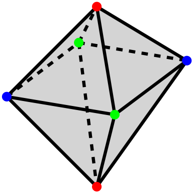
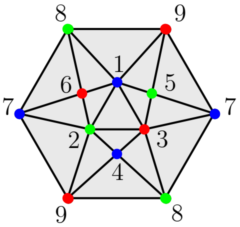
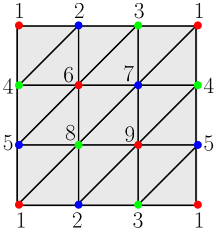
The problem

Definition: A d-dimensional simplicial complex is **balanced** if its graph can be properly $(d+1)$-colored.







The **barycentric subdivision** of any triangulation is balanced, with a large vertex set.

Problem: Find balanced triangulations of d-manifolds on **few vertices**.

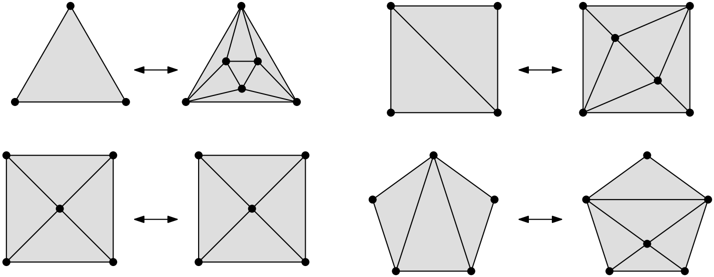
Our solution

Reduce the number of vertices using local transformations.

Definition: (Izemstiev-Klee-Novik, 2017) A **cross-flip** on a balanced *d*-dimensional simplicial complex Δ is given by:

$$\Delta \to (\Delta \setminus \Gamma) \cup (\partial \mathcal{C}_{d+1} \setminus \Gamma),$$

for some induced subcomplex $\Gamma \subset \Delta$ that is isomorphic to a shellable and coshellable *d*-dimensional subcomplex of $\partial \mathcal{C}_{d+1}$.



We obtain results for $d = 2, 3$. Already for $d = 3$ it is hard to find a sequence of flips reducing the number of vertices.

$\|\Delta\|$	Min $f(\Delta)$	$f(\mathrm{Bd}(\Delta))$	Min. Balanced f known
S^2	4	14	6
$\mathbb{T}$	7	42	9
$\mathbb{T}^{\#2}$	10	70	12
$\mathbb{T}^{\#3}$	10	80	14
$\mathbb{T}^{\#4}$	11	96	14
$\mathbb{T}^{\#5}$	12	112	16
$\mathbb{R}\mathbf{P}^2$	6	31	9
$(\mathbb{R}\mathbf{P}^2)^{\#2}$	8	48	11
$(\mathbb{R}\mathbf{P}^2)^{\#3}$	9	59	12
$(\mathbb{R}\mathbf{P}^2)^{\#4}$	9	64	12
$(\mathbb{R}\mathbf{P}^2)^{\#5}$	9	69	13

$\|\Delta\|$	Min $f_0(\Delta)$	$f_0(\mathrm{Bd}(\Delta))$	Min. Bal. f_0 known.
S^3	5	30	8
$S^2 \times S^1$	10	148	14
$S^2 \underline{\times} S^1$	9	126	12
$\mathbb{R}\mathbf{P}^3$	11	182	16
$L(3, 1)$	12	240	16
$L(4, 1)$	14	308	20
$L(5, 1)$	15	358	22
$L(5, 2)$	14	316	20
$L(6, 1)$	16	408	24
$(S^2 \times S^1)^{\#2}$	12	208	16
$(S^2 \underline{\times} S^1)^{\#2}$	12	208	16
$(S^2 \times S^1)\#\mathbb{R}\mathbf{P}^3$	14	264	20
$(\mathbb{R}\mathbf{P}^3)^{\#2}$	15	314	21
$(S^2 \times S^1)^{\#3}$	13	262	20
$(S^2 \underline{\times} S^1)^{\#3}$	13	262	19
$S^1 \times S^1 \times S^1$	15	390	24
Oct. space	15	378	24
Cube space	15	330	23
Poincaré	16	392	26
$\mathbb{R}\mathbf{P}^2 \times S^1$	14	308	24
Non-shellable	18	536	28
Shellable, non-vd	16	400	22