Unique Rectification in d-complete Posets: Towards the K-theory of Kac-Moody Flag Varieties

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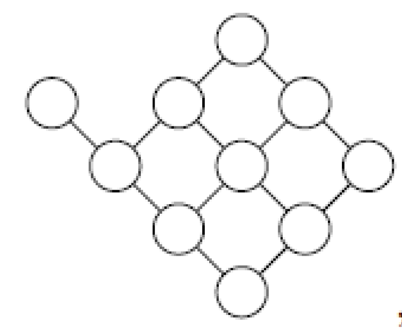
Background

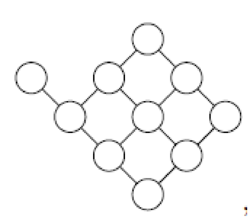
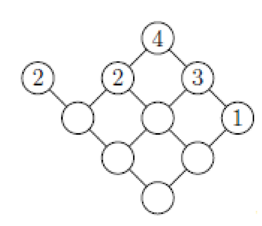


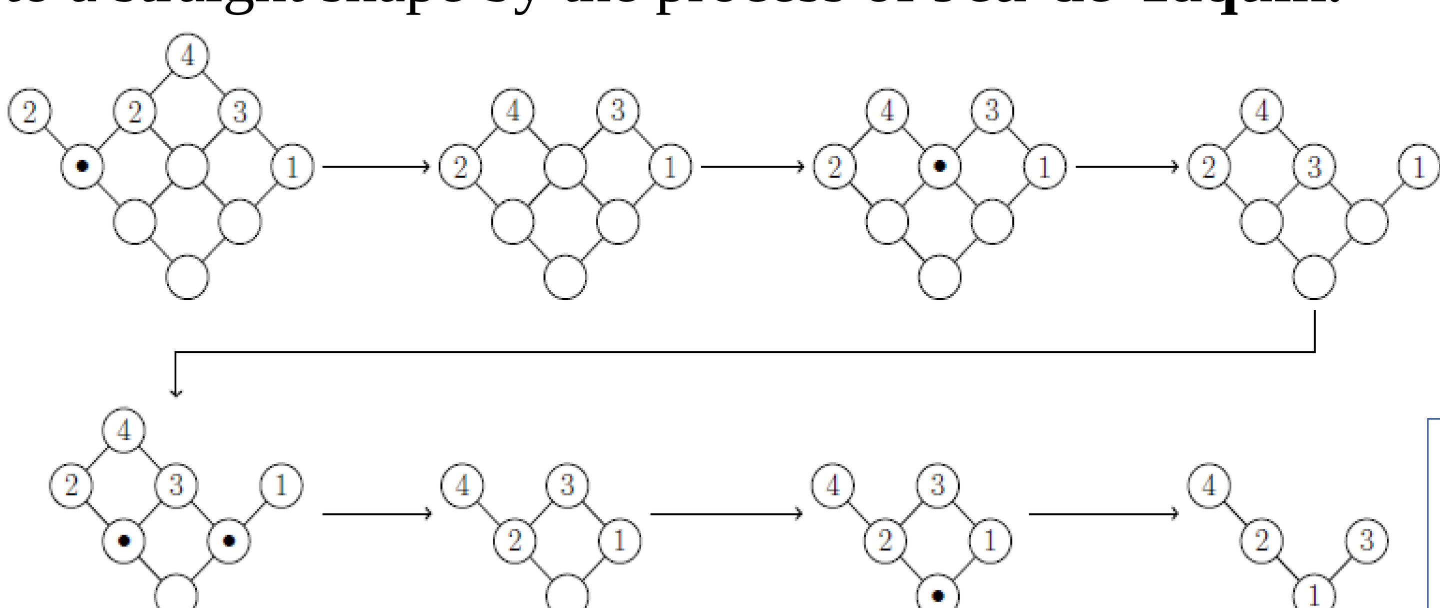
Definitions

Consider a finite, connected poset $\mathcal{P}$.

A straight shape is a downwardly closed subset of $\mathcal{P}$.
Given two straight shapes $\lambda \subseteq \nu \subseteq \mathcal{P}$, an **increasing tableau** of shape $\nu \setminus \lambda$ is a strictly order-preserving labelling of $\nu \setminus \lambda$.
That is, $x < y$ implies $T(x) < T(y)$.

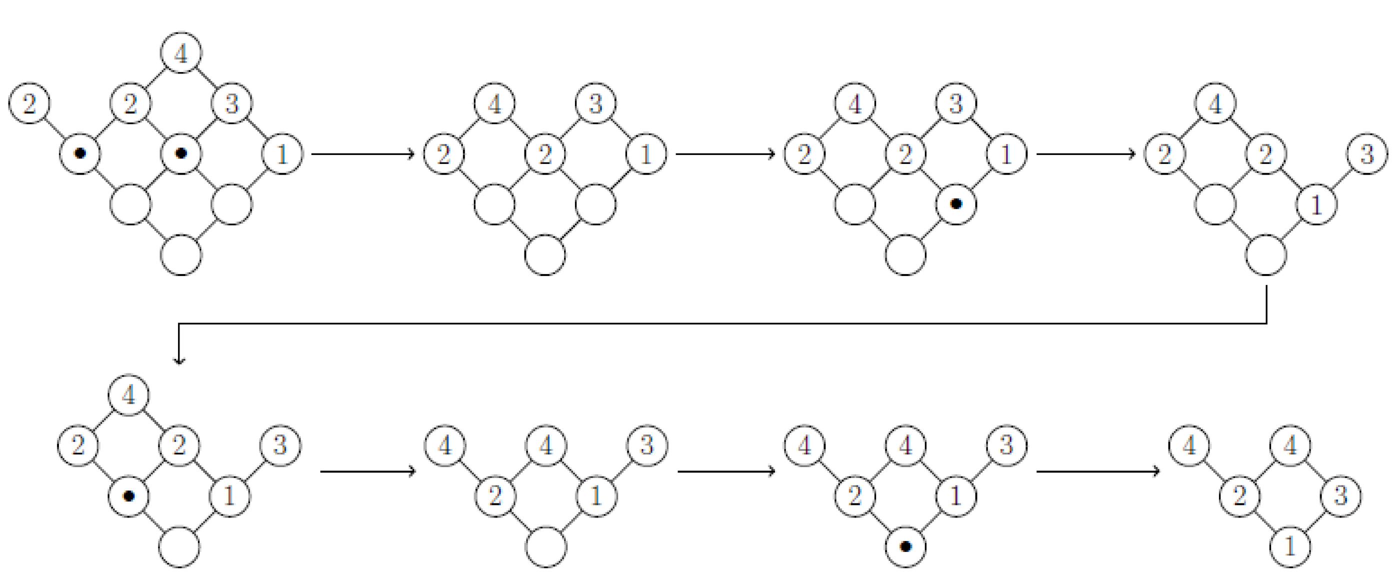
Rectification

Example: Let $\mathcal{P}$ =  , T =  . We rectify this tableau to a straight shape by the process of **Jeu-de-Taquin.**



Not all tableau rectify uniquely!

But also:



Background

A tableau U is a **unique rectification target (URT)** if any tableau which *can* rectify to U, *always* rectifies to U.

Say a poset $\mathcal{P}$ has **enough URTs** if there is a URT of shape μ for every straight shape μ of $\mathcal{P}$.

Theorem (Thomas-Yong 2009, Buch-Samuel 2016)

Minuscule posets have enough URTs.

Theorem (Thomas-Yong 2009, Buch-Samuel 2016)

For any minuscule variety, $K^{\nu}_{\lambda,\mu} = (-1)^{|\nu|-|\lambda|-|\mu|} \cdot \#$ *increasing tableaux of shape* ν/λ *that rectify to any fixed URT of shape* μ.

Question: (Thomas-Yong 2009, Buch-Samuel 2016) Do d-complete posets have enough URTs?

Conjecture (Ilango-P-Zlatin 2018)

Yes!

We also conjecture a similar combinatorial formula for the structure constants of Λ-minuscule varieties of Kac-Moody spaces.

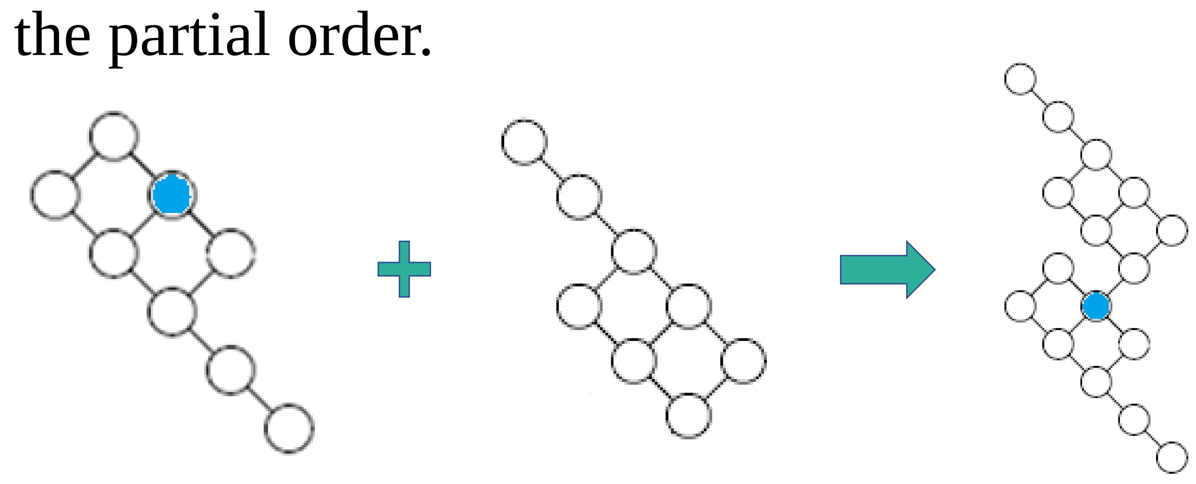
Conjecture (Ilango-P-Zlatin 2018)

For λ, μ, ν *all* Λ*-minuscule,* $K^{\nu}_{\lambda,\mu} = (-1)^{|\nu|-|\lambda|-|\mu|} \cdot \#$ *increasing tableaux of shape* ν/λ *that rectify to any fixed URT of shape* μ.

Results

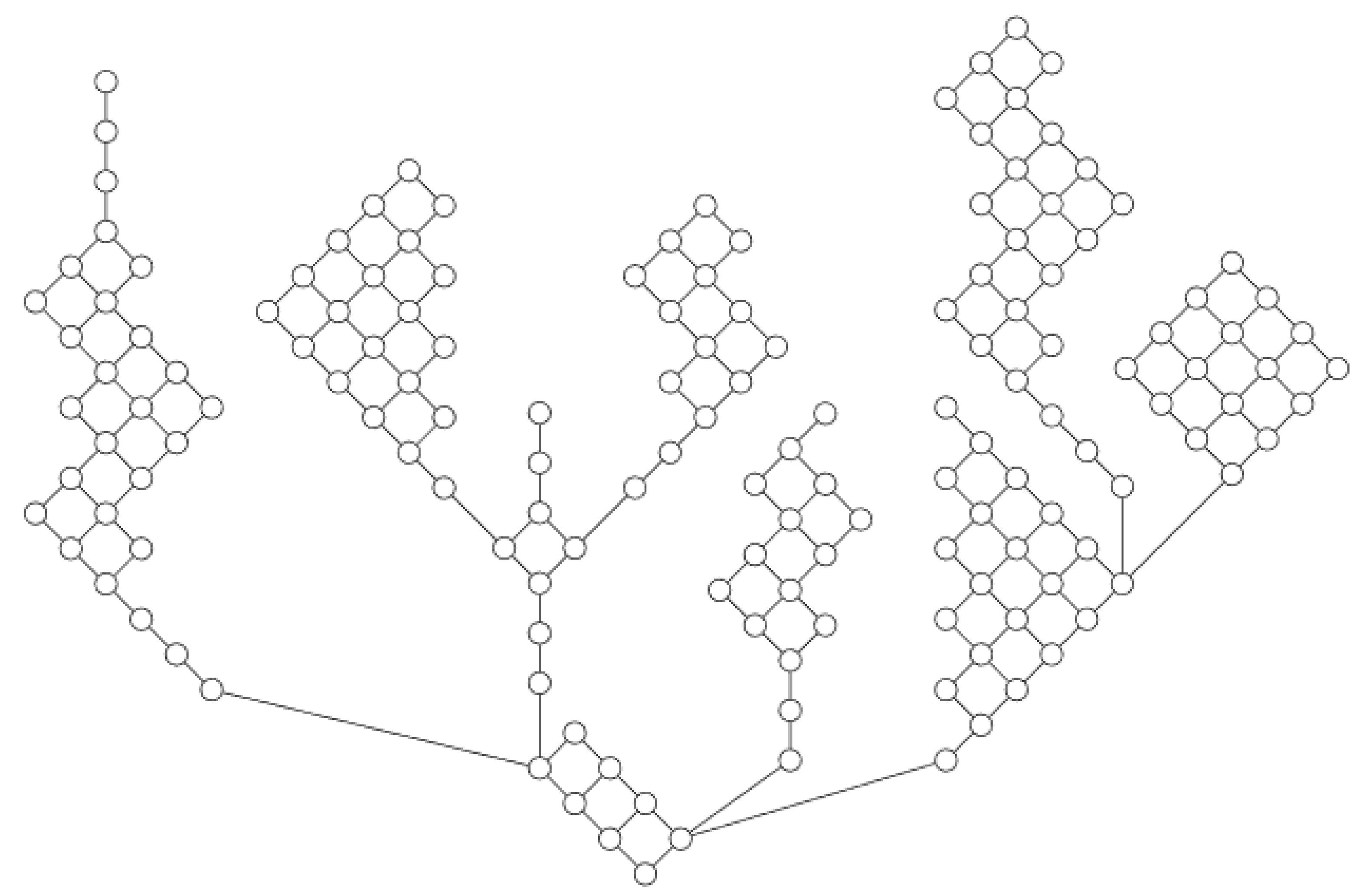
A combinatorial characterization of *d*-complete posets

Let $\mathcal{P}, \mathcal{Q}$ be posets and let $p \in \mathcal{P}$. Then the **slant sum** of $\mathcal{Q}$ onto p is the poset obtained by sticking the minimum of $\mathcal{Q}$ directly above p in the partial order.



Say a poset is **irreducible** if it is not the slant sum of two d-complete posets.

R. Proctor: all d-complete posets can be uniquely decomposed as the slant sum of the 15 families of irreducible components. (1999)



Theorem: This poset has enough URTs!

Results

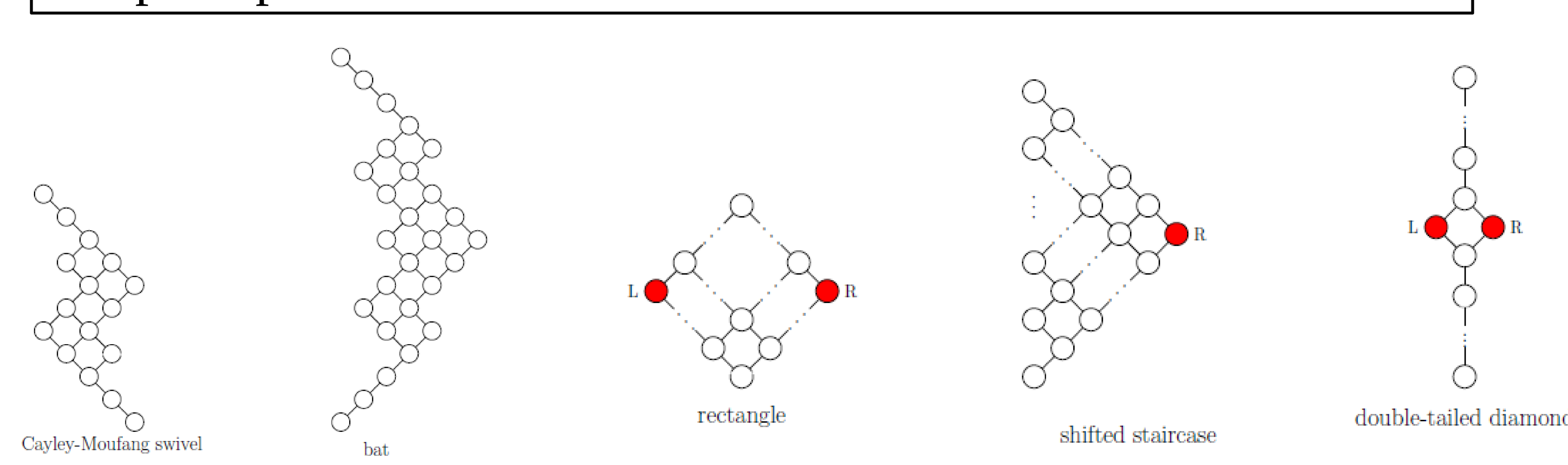
Method of Attack

1) Show that each irreducible component has enough URTs
2) Show that this property is preserved by the slant sum operation.

We define a p-chain URT to be a URT which is still a URT after a chain of any length is slant summed onto p.

Using p-chain URTs, we show (2) for 5 out of 15 of the irreducible families.

We observe that every minuscule poset is an irreducible d-complete poset in one of these 5 families.



Buch-Samuel showed (1) for minuscule posets in 2016.
Therefore, (1) and (2) hold for minuscule posets.

We conclude that any poset built out of slant sums of minuscule posets has enough URTs.

In fact, we can explicitly describe the URTs for each straight shape. They are **minimally labelled.**

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