**Background**

The set of $k$-dimensional subspaces of $C^k$ is called the Grassmannian. Modern Schubert calculus aims to study its geometry through its (connective) $K$-theory ring. The Schubert structure sheaf basis is represented by symmetric Grothendieck polynomials $e_\lambda(x)$. A. Buch has shown that $e_\lambda(x)$ can be given as a sum over set-valued tableaux $SV^\lambda$. Buch's uncracking bijection gives a crystal structure where $SV^\lambda$ realises the representation of some quantum group. One application of our proposed $K$-crystal structure is to construct a $K$-theoretic analog of crystals on set-valued tableaux.

**Crystal structure**

Let $T \in SV^\lambda$ and $i \in \{1, \ldots, n\}$. Write $w = \ell(T)$ and $\ell(T) = (i_1, \ldots, i_{\ell(T)})$ where $\ell(T) = \ell(T)$. A. Buch's uncracking bijection gives a crystal structure where $SV^\lambda$ realises the representation of some quantum group. One application of our proposed $K$-crystal structure is to construct a $K$-theoretic analog of crystals on set-valued tableaux.

**Weak $K$-crystal example**

Let $U$ and $T$ be set-valued tableaux of shape $\lambda$ and $\mu/\lambda$, respectively. We define operators $h_i$ on set-valued tableaux whose entries are in an alphabet $\mathcal{A}$. $h_i$ is defined by $h_i(u) = u$ for all $i \in \mathcal{A}$, which gives the set of all $\lambda$-tableaux with a given shape. We define operators $h_i$ on set-valued tableaux whose entries are in an alphabet $\mathcal{A}$. $h_i$ is defined by $h_i(u) = u$ for all $i \in \mathcal{A}$, which gives the set of all $\lambda$-tableaux with a given shape.

**Rectification**

Let $U$ and $T$ be set-valued tableaux of shape $\lambda$ and $\mu/\lambda$, respectively. We define operators $h_i$ on set-valued tableaux whose entries are in an alphabet $\mathcal{A}$. $h_i$ is defined by $h_i(u) = u$ for all $i \in \mathcal{A}$, which gives the set of all $\lambda$-tableaux with a given shape.

**Open problems**

- Construct a weak $K$-crystal structure on $SV^\lambda$.
- Determine a tensor product rule such that the result categorifies the $K$-theory ring of the Grassmannian.
- Determine a K-crystal insertion rule such that $K$-rectification agrees with Buch's insertion algorithm.
- Determine a combinatorial rule that characterizes the set-valued tableaux that appear in $SV^\lambda$ for a given weak $K$-crystal structure.