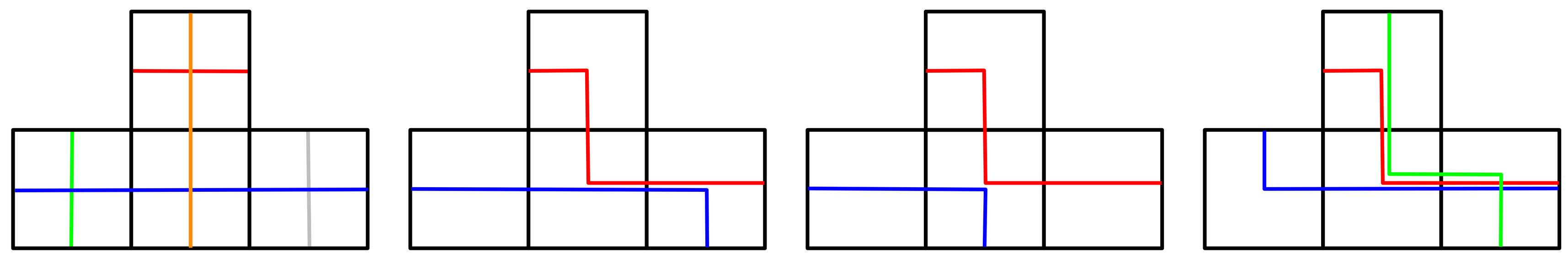
Non-kissing — vs — Non-crossing

Yann Palu (UPJV) jtw Vincent Pilaud (LIX, CNRS) and Pierre-Guy Plamondon (Orsay)

From walks on a grid [McConville] ...



The straight walks A kiss A shy kiss No kissing

Walk = NW to SE maximal path in a grid of any (fixed) shape.

A walk ω kisses a walk ω' if

- ω and ω' share a common subpath ρ,
- ω enters ρ from W and leaves it towards S,
- ω' enters ρ from N and leaves it towards E.

(Reduced) non-kissing complex = clique complex of (non-straight) walks for the compatibility relation of non-kissing.

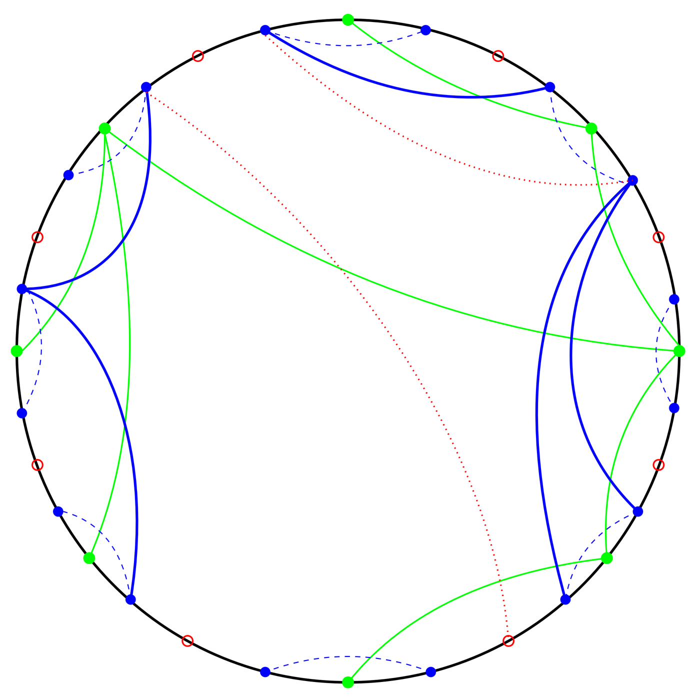
Thm [McC]: *The reduced non-kissing complex is pure and thin.*

From accordions on a disk [Garver–McC] ...

Exm:

- a dissection (green),
- some accordions (blue),
- some non-accordions (red).

Accordions not crossing any other accordion are dotted blue.



(In the language of [Manneville–Pilaud])

Vertices of alternating colour green-blue-red-blue.

Dissection = collection of non-crossing green diagonals.

Accordion = blue diagonal crossing a connected subset of the dissection (including boundary segments).

(Reduced) non-crossing complex = clique complex of (non-dotted) accordions of a fixed dissection for the compatibility relation of non-crossing.

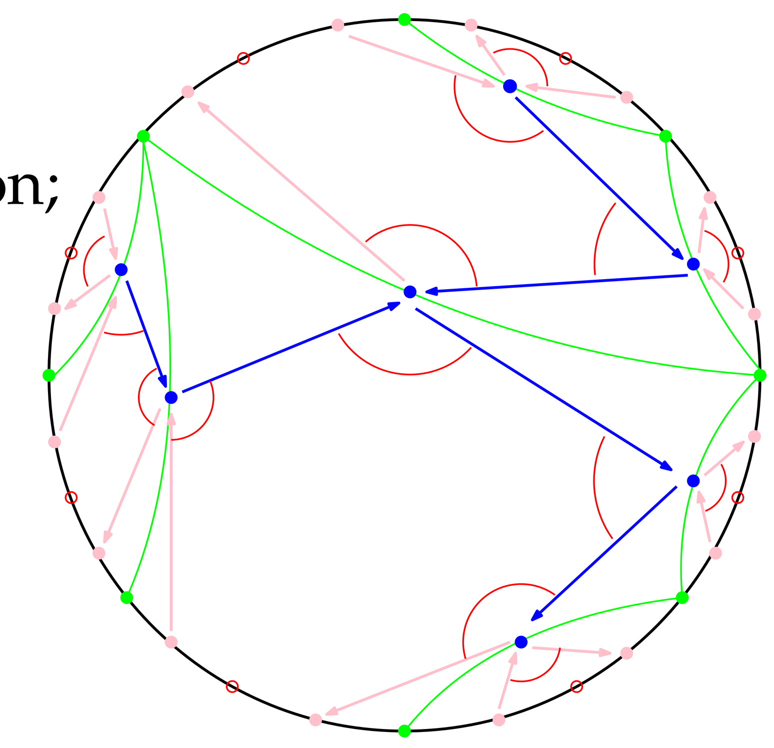
Thm [G-McC]: *The reduced non-crossing complex is pure and thin.*

From dissections to gentle bound quivers

Quiver of a dissection:

- Vertices = arcs of the dissection;
- Arrows = angles;
- Relations = two consecutive arrows in a same cell.

Take boundary segments into account to obtain the blossoming quiver.



Prop: *The bound quiver associated with any dualizable dissection of a marked surface is gentle.*

Main Results [PPP]

Thm: *Gentle bound quivers* $\longleftrightarrow$ *Marked surfaces endowed with a pair of dual dissections.*

(see also [Baur–Coelho-Simoes])

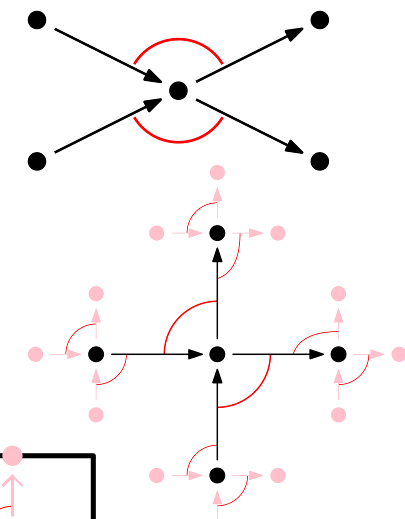
Thm: *Walks on the quiver Q* $\longleftrightarrow$ *Accordions on the surface S*

Kissing $\longleftrightarrow$ *Crossing*

Non-kissing complex of Q $\longleftrightarrow$ *Non-crossing complex of S.*

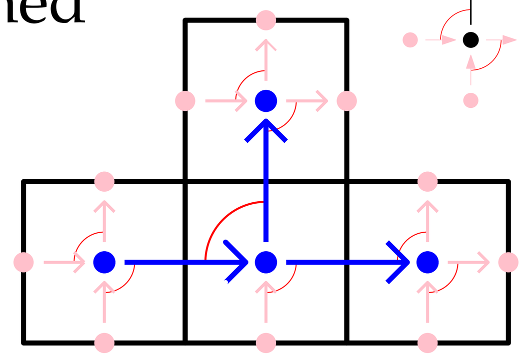
... to walks on a gentle bound quiver [PPP]

Gentle bound quiver Q = oriented graph with relations (forbidden paths) of lenght 2; obtained by glueings of a local configuration. Moreover, any vertex must have out-degree at most 2 and in-degree at most 2.



Blossoming bound quiver $Q^{❀}$ = obtained by making each (old) vertex 4-valent.

Exm. The gentle bound quiver of a grid, and its blossoming quiver:

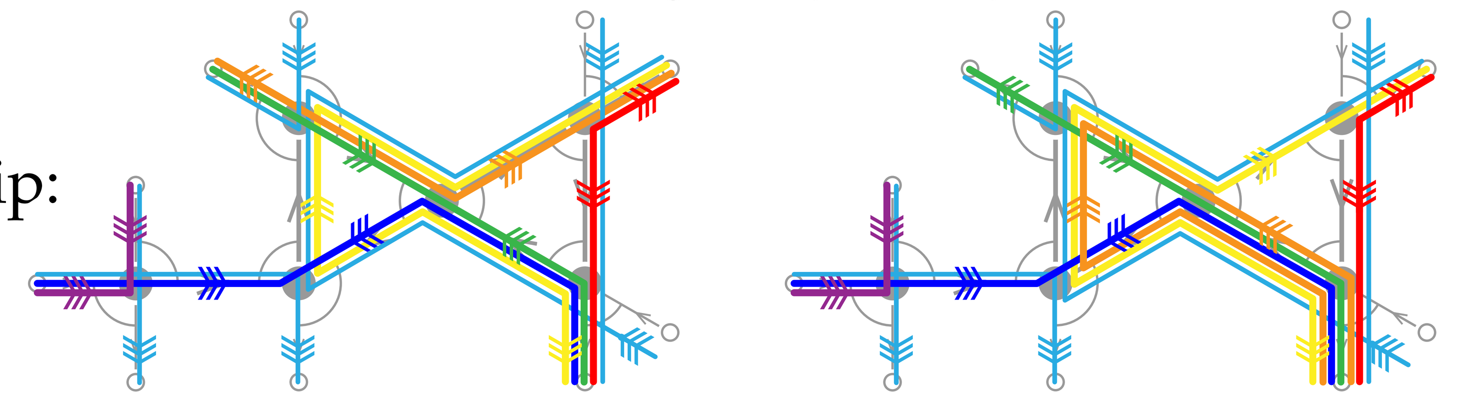


String of Q = word on arrows and their inverses not containing any $\alpha\alpha^{-1}$, $\alpha^{-1}\alpha$ nor any $\alpha\beta$, $\beta^{-1}\alpha^{-1}$ if $\alpha\beta$ is a relation.

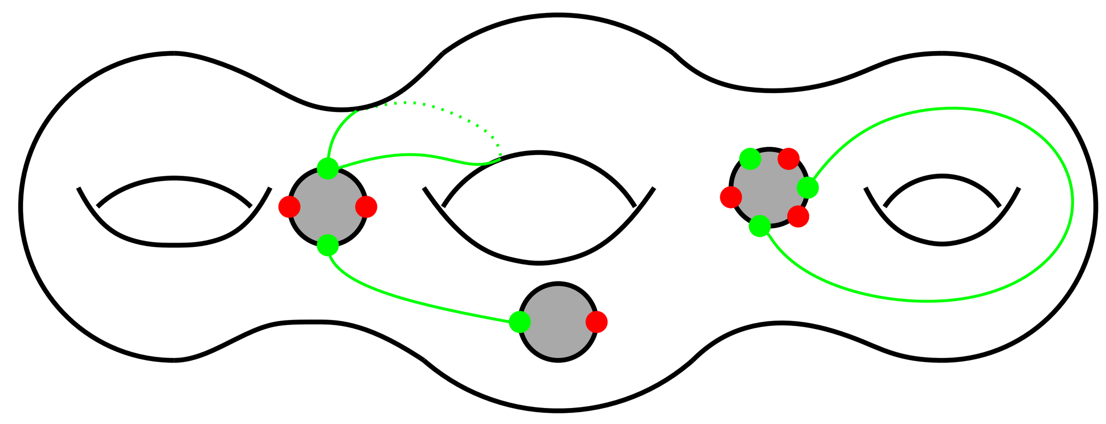
Walk on Q = maximal string of $Q^{❀}$ (thus blossom to blossom).

Thm [PPP]: *The reduced non-kissing complex is pure and thin.*

Exm of flip:

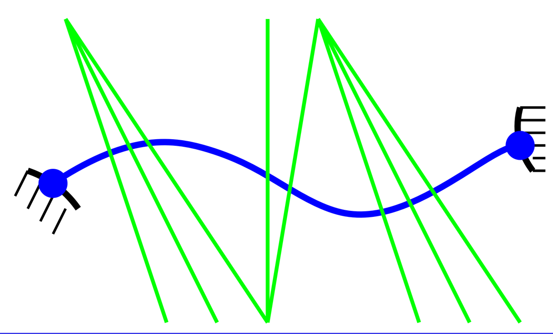


... to accordions on a Riemann surface [PPP]



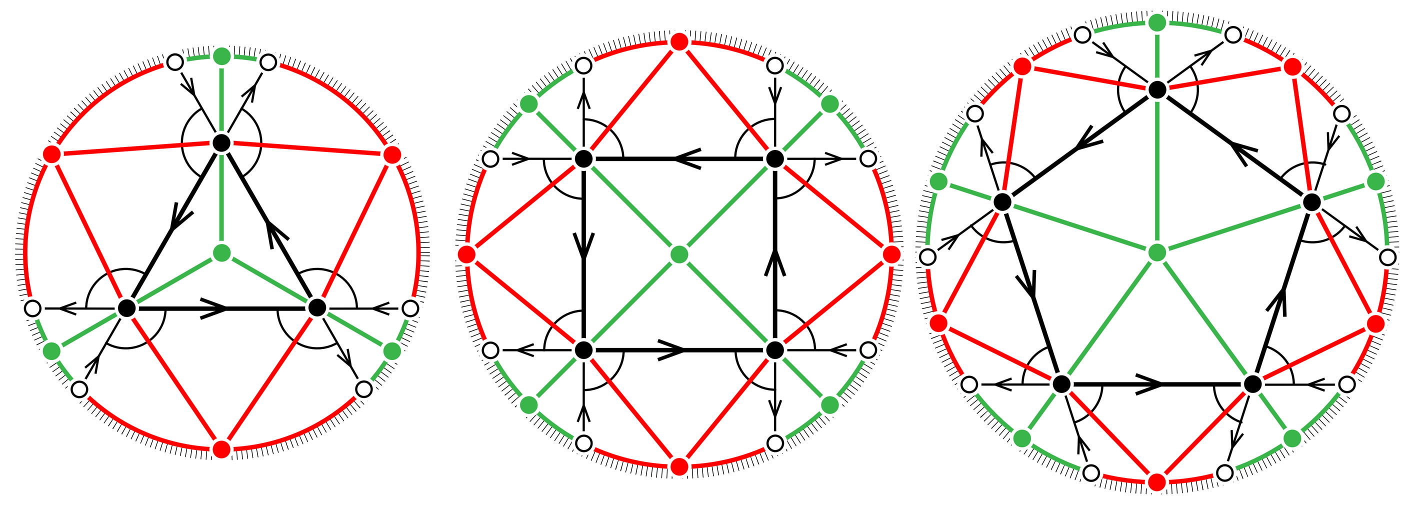
Some arcs on a Riemann surface with boundary and marked points.

Given a dualizable (= every cell of the dissection contains exactly one red vertex, possibly not on the bounday) dissection of a Riemann surface, can define accordions.



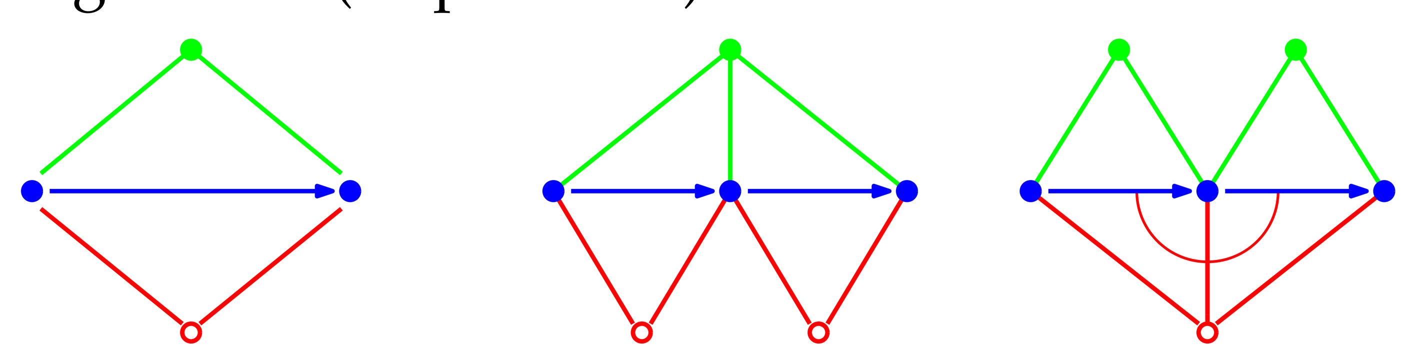
Thm [PPP]: *The reduced non-crossing complex is pure and thin.*

From gentle bound quivers to dissections



The surface of three cyclic quivers

Given a gentle bound quiver Q, take its blossoming, then glue a green (resp. red) triangle to the left (resp. right) of each arrow. Glue two consecutive red (resp. green) triangles if the corresponding arrows (resp. do not) form a relation.



Prop: *The construction above yields a marked surface endowed with a pair of dual dissections, given by green / red arcs.*

Relation with representation theory

arXiv:1707.07574 – NKC and tau-tilting for gentle algebras.

arXiv:1807.04730 – NKC and NCC for locally gentle algebras.

yann.palu@u-picardie.fr