**Background**

One of the gems of 20th-century mathematics is the theory of symmetric functions and symmetric polynomials. Interpreting Schur functions through the cohomology of Grassmannians leads one to consider K-theoretic analogues of the classical bases. Additionally, we wish to lift the theory of symmetric polynomials to larger rings of quasisymmetric and asymmetric polynomials.

We introduce two new bases of $ASym_n = \mathbb{Z}[x_1, \ldots, x_n]$. The quasiLascoux basis is a K-theoretic deformation of the quasikyel basis that is also an asymmetric lift of quasiGrotendieck polynomials, a refinement of the Lascoux basis, and a simultaneous coarsening of the glide and Lascoux atom bases. Kaons are K-theoretic deformations of pions that are simultaneous refinements of glides and Lascoux atoms.

**Theorem 2:**

Given a set-valued filling $F$ of a skyline diagram of shape $a$ and weight $w(F)$, the quasiLascoux polynomials $\mathcal{L}_a$ can be obtained from $\mathcal{L}_{\mathcal{F}}$ by the following general formula:

$$\mathcal{L}_a = \sum_{F} w(F) \mathcal{L}_{\mathcal{F}}$$

**Definition 1:**

For a weak composition $a$, the quasiLascoux polynomial $\mathcal{L}_a$ is a positive sum of multi-fundamental polynomials $\mathcal{L}_a = \sum_{\mathcal{F} \in \mathcal{F}[a]} \mathcal{L}_{\mathcal{F}}$, where $\mathcal{F}[a]$ is a set of all possible skyline diagrams of shape $a$.

**Theorem 2:**

Each quasiGrotendieck polynomial $\mathcal{G}_a \in QSym_n$ is a positive sum of multi-fundamental polynomials $\mathcal{G}_a = \sum_{\mathcal{F} \in \mathcal{F}[a]} \mathcal{G}_{\mathcal{F}}$, where $\mathcal{F}[a]$ is a set of all possible skyline diagrams of shape $a$.

**Conjecture 1:**

For a weak composition $a$, $\sum \mathcal{M}_a \in \{0, 1\}$ and $\sum \mathcal{G}_a \in \{0, 1\}$, where both sums are over all weak compositions $a$.

**Conjecture 2:**

For a weak composition $a$, $\mathcal{L}_a \in \{0, 1\}$. $\mathcal{L}_a$ is a positive sum of Lascoux atoms.

**Conjecture 3:**

For any weak compositions $a$ and $b$, the product $\mathcal{L}_a \cdot \mathcal{L}_b$ of a kaon and a glide polynomial expands positively in kaons.

**Conjecture 4:**

$(\text{Reiner}, \text{Shimozono}, \text{Yong})$ Each quasiGrotendieck polynomial $\mathcal{G}_a$ is a positive sum of Lascoux polynomials $\mathcal{L}_b$.

**References**


